

acoustic faces (see Fig. 2), the complete set of three independent elastic constants $C \equiv C_{44}$, $C' \equiv 1/2 (C_{11} - C_{12})$ and $C_{11}' \equiv C_{11} - C' + C$, are given directly by ρv_i^2 where the v_i for the three constants C , C' and C_{11}' are the values for (i) a transverse wave with particle motion along the [001] direction (C), (ii) a transverse wave with particle motion along [110] (C') and (iii) a longitudinally polarised wave (C_{11}'). Then the adiabatic bulk modulus $B_s = C_{11}' - C_{44} - 1/3 C'$. For the usual metallic anisotropies, $C > C'$, somewhat greater precision of the measurement of B_s may be obtained by use of a second crystal with [100] orientation. Then $C_{11} = \rho v^2$ for the longitudinal wave, and $B_s = C_{11} - 4/3 C'$, usually a relatively small correction from the direct measurement, e.g. about 20% in the case of silver, 12% in sodium, is necessary. The acoustic wave velocities are of course the quotients of twice the length of the sample between acoustic faces (L), and the transit time t of a pulse, measured with the calibrated sweep delay, i.e.

$C = \rho 4L^2/t^2$. If the pressure is changed, ρ , L and t change, yielding $\frac{d \ln C}{dP} = + \frac{1}{3B_T} - \frac{2}{t} \cdot \frac{dt}{dP}$

for small changes. The first term arises from changes of L and ρ with pressure. The isothermal bulk modulus must be computed from the measured adiabatic bulk modulus from the thermodynamic relation

$$B_s = B_T \left(1 + \frac{TV\beta^2 B_T}{Cv} \right)$$

where β is the volume coefficient of thermal expansion, T the absolute temperature, Cv and V the heat capacity and volume per mole. The second term is given by the data taken, i.e. *change* of transit time with pressure. In our technique the change is always measured with respect to a nearby time marker to reduce any effects due to time zero drift. Figs. 3 and 4 show typical curves of change of time of echo arrival relative to a nearby time marker *versus* change of the coil resistance of the pressure gauge, the latter being proportional to pressure change. [Note the complete absence of hysteresis between points taken with increasing pressure and those taken with decreasing pressure. This is important because it is contrary to the assumption that length hysteresis is a natural phenomenon in most samples, even if the pressure is truly hydrostatic.⁶ The assumption is based on observation of apparent hysteresis of length in compression determinations made with lever piezometers, and seems to indicate the presence of friction somewhere in the length-measuring apparatus.] From these data and values of the constants at zero pressure, we

calculate the quantities dC/dP or the more characteristic quantity $d \ln C/d \ln r = - \frac{3B_T}{C} \cdot \frac{dC}{dP}$

which is given the title the 'logarithmic hydrostatic strain derivative' or just 'strain derivative'. The precision of such measurements has recently been investigated extensively by Corll and Smith at Case Institute on single crystals of silver, with orientations [100], [110] and [111], confirming earlier estimates of about 0.3% in the C , 3% in the derivatives.³⁰ The precision will of course be higher in the more compressible materials for which the change in the stiffness is large over the pressure range used; and lower for materials less compressible than silver. This precision is we think remarkable and implies that the use of ultrasonic methods to obtain equations of state of solids deserves serious consideration, yet the only serious work in such a direction is that of Anderson¹² on fused silica. Our own measurements have been completely aimed at finding derivatives, and attempting to evaluate them at zero pressure. Thus, in the case of sodium, the data taken were arbitrarily confined to a pressure range less than 2kb and no special attempt was made

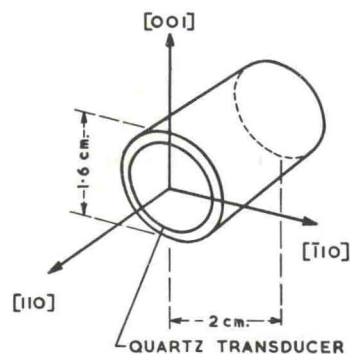


Fig. 2. Typical specimen size and shape
X- or Y-cut quartz crystal transducers
generate longitudinal or transverse
waves respectively

to interpret the data in terms of $\Delta V/V$ over a wide range of pressures. Measurements taken over the more extended range to 7.8 kb revealed no difficulty (see Fig. 4), and indeed no real problem should appear in extending such measurements to a much higher pressure or to higher or lower temperatures. The employment of a larger model of the Swenson modification³¹ of the high-pressure apparatus of Dugdale & Hulbert³² for use at temperatures near 4.2°K should even permit very accurate PV measurements to be made acoustically (in materials which do not have structure transformations) in this temperature range. As will be seen in the third section of this paper, the breakdown of the assumption that Poisson's ratio is independent of pressure is very serious in discussions of the lattice contribution to the thermal expansion. Yet, if one uses $P-V$ data alone as Slater was forced to do (for lack of shear data), no knowledge of pressure dependence of the Poisson ratios is obtained. Further, regarding accuracy of measurements ΔV vs P reference is made to pp. 83—85 of Swenson's recent review article.⁶ We agree completely that the best ΔV vs P data can be provided by careful acoustic measurements. A discussion of the use of the acoustic method for determining parameters of the equation of state and values of thermodynamic functions, is given by Anderson.¹² It is suggested that those who contemplate highly refined $P-V-T$ measurements on solids, at least for those cases where single crystals may be obtained, should consider whether extension of this method to low temperatures using techniques perfected by Swenson⁶ or to high temperatures using a stepped crystal as has been done by Bernstein,³³ would not produce results sufficiently more reliable than those obtained by direct compression of polycrystalline materials to warrant the additional effort required.

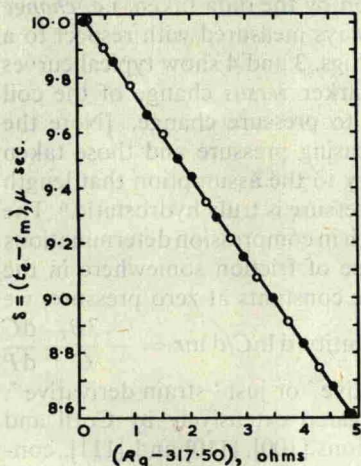
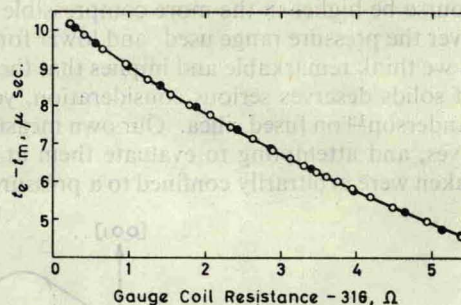


Fig. 3. Typical data plot showing difference between the time, t_e , of arrival of one of the maxima of echo No. 7 of the longitudinal wave in a 3.164 cm. long aluminium crystal and a fixed time-marker, t_m , as a function of pressure-gauge coil resistance, R_g

● increasing pressure ○ decreasing pressure

Fig. 4. Change with pressure of the time of arrival of an echo for the C_{44} wave in sodium

The pressure range indicated is about 7800 bars



(2) Interpretation of data

To introduce the subject of interpretation of the data obtained in terms of lattice interactions, consider Table I which gives values of the elastic constants and their strain derivatives for a variety of materials.

The measurements have all been made at room temperature. Values for germanium were measured by McSkimmin at the Bell Laboratories and for rubidium iodide at the Princeton University and the remainder at the Case Institute of Technology. Note that dimensionally elastic constants have the form e^2/r^4 where e is the electron charge and r a linear dimension.